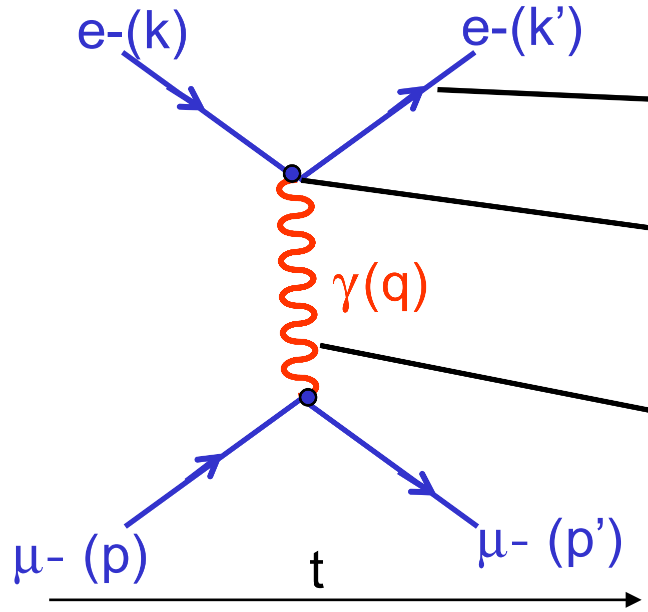
We have seen a lot of discoveries, new ideas, concepts…

- Discovery of charged leptons and neutrinos (total of 6)
- Discovery of quarks… u, d,s,c,b,t (total of 6)
- We have seen that we need three interactions to describe different cross section , lifetimes… : strong, weak, electromagnetic
- We have introduced the new way of looking at the interactions as exchange of virtual particles (bosons) : photons, gluons, W, Z…
- We have seen that there are quantum numbers which are conserved in certain interactions and not in others…
- In particular we have seen that Parity is not conserved in weak interaction and that the fondamental fermions are left handed (for massless particles)
- We have still to see that the forces/interactions in modern quantum field theory comes from a symmetry principle (local gauge transformation)

Feynman rules :



External lines: fields (spinors, vectors, …)

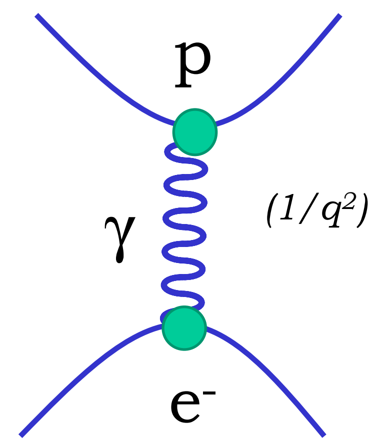
Vertex: $\sqrt{\alpha}$ factor in the matrix element « interaction intensity »

Propagator:

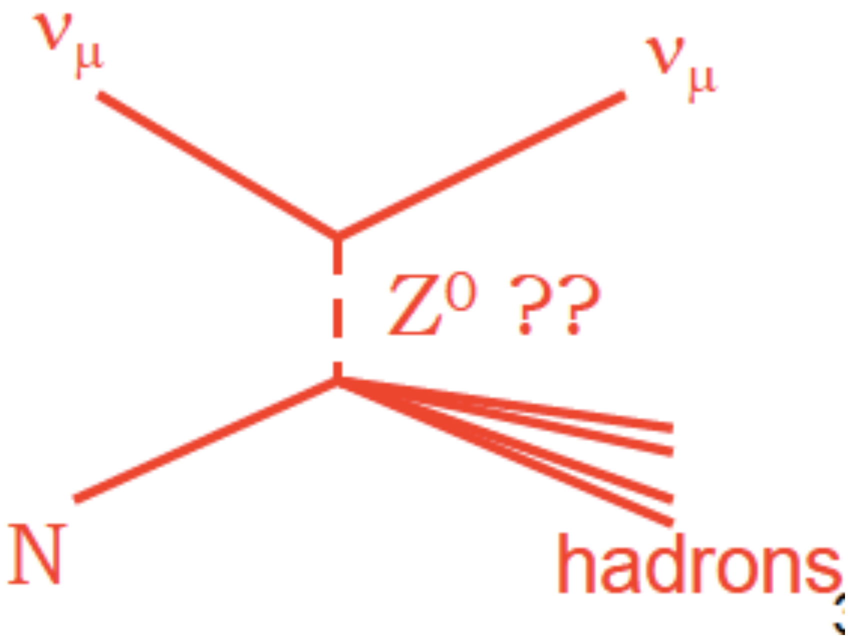
factor $ig_{\upsilon\nu}/(q^2-m^2)$ (depends also on spin …)

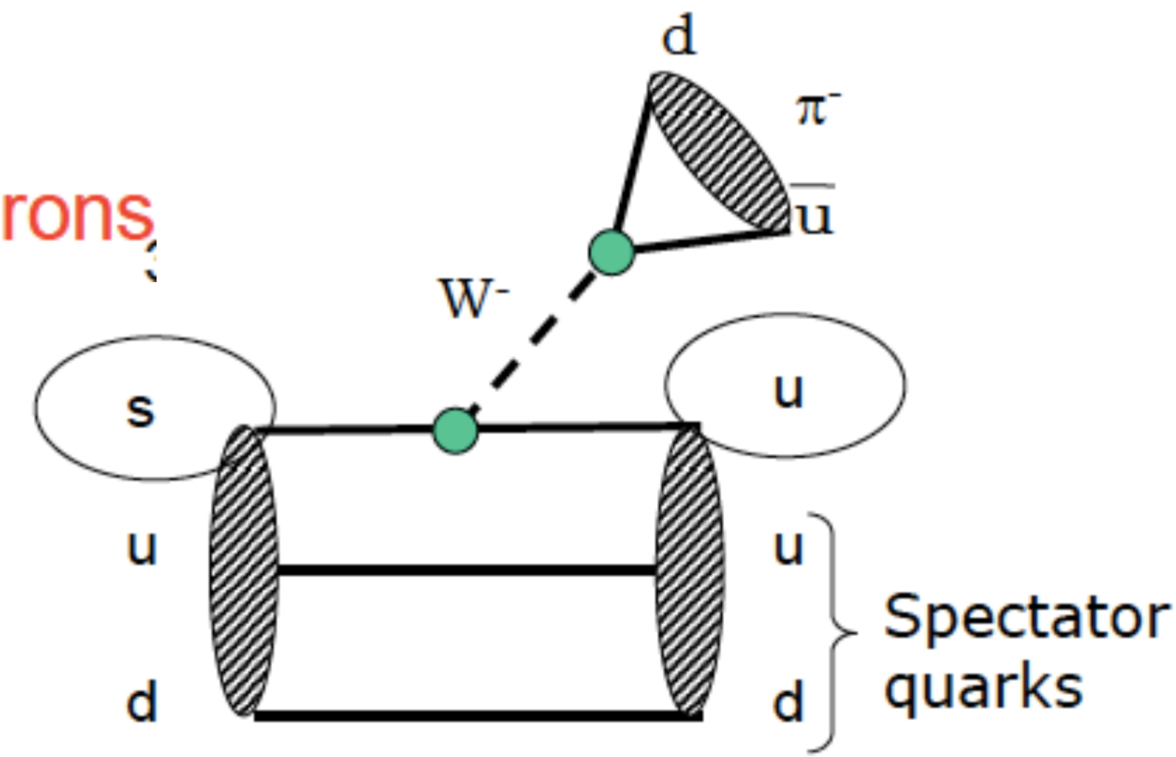
$$M = \left(e\bar{u}_p \gamma^\mu u_p\right)\left(-\frac{1}{q^2}\right)\left(-e\bar{u}_e \gamma_\mu u_e\right)$$

$$M = -\frac{e^2}{q^2}\left(J_\mu\right)_p \left(J^\mu\right)_e$$



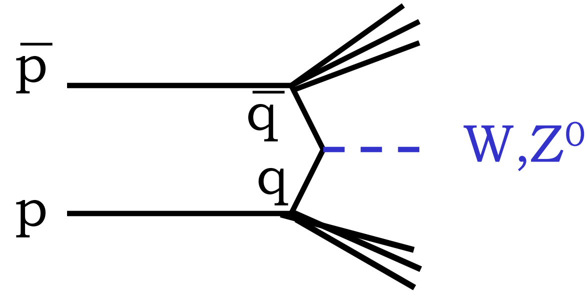
And we have seen that



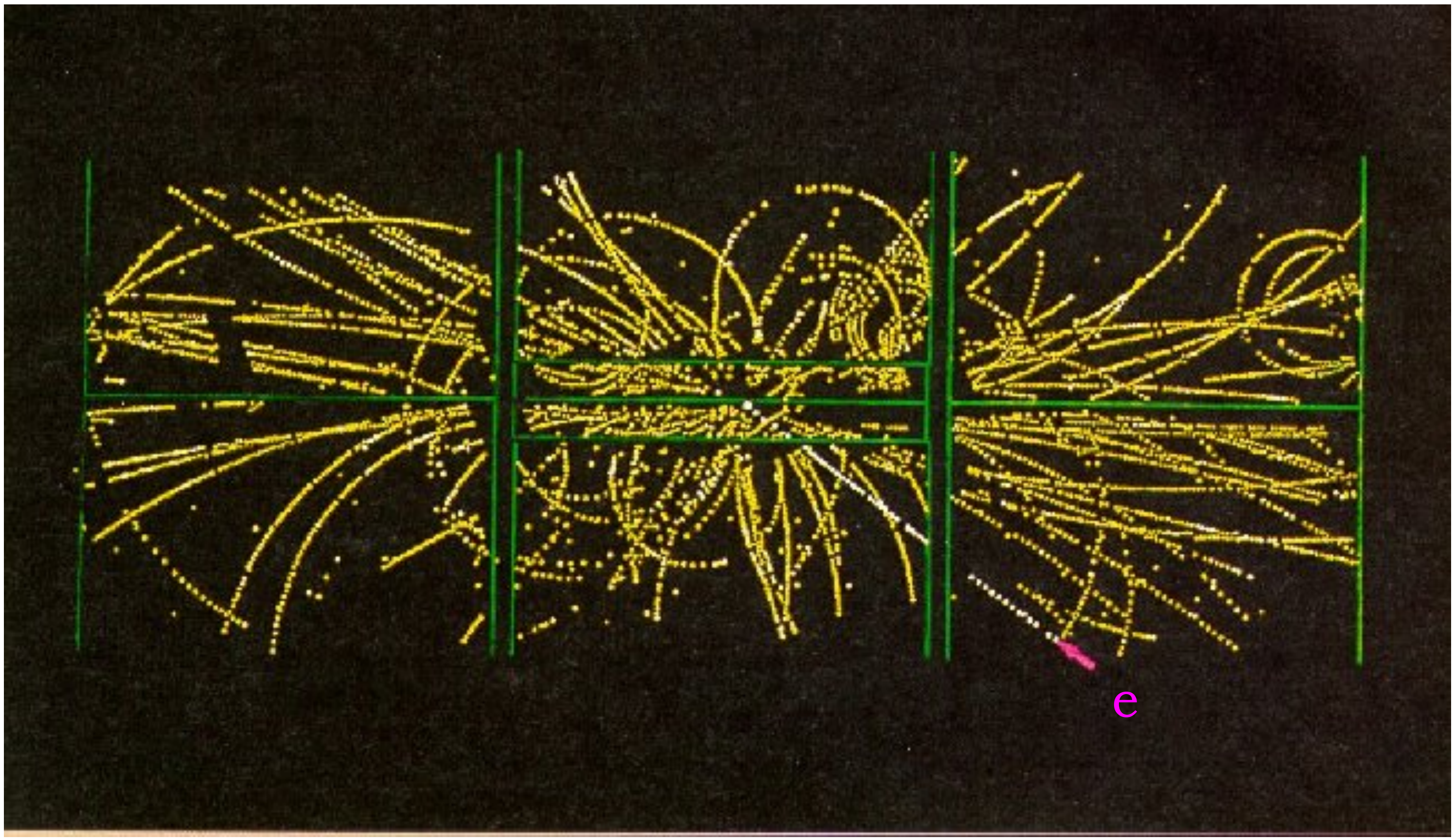


Discovery of $W^{\pm}$ and of the Z^0

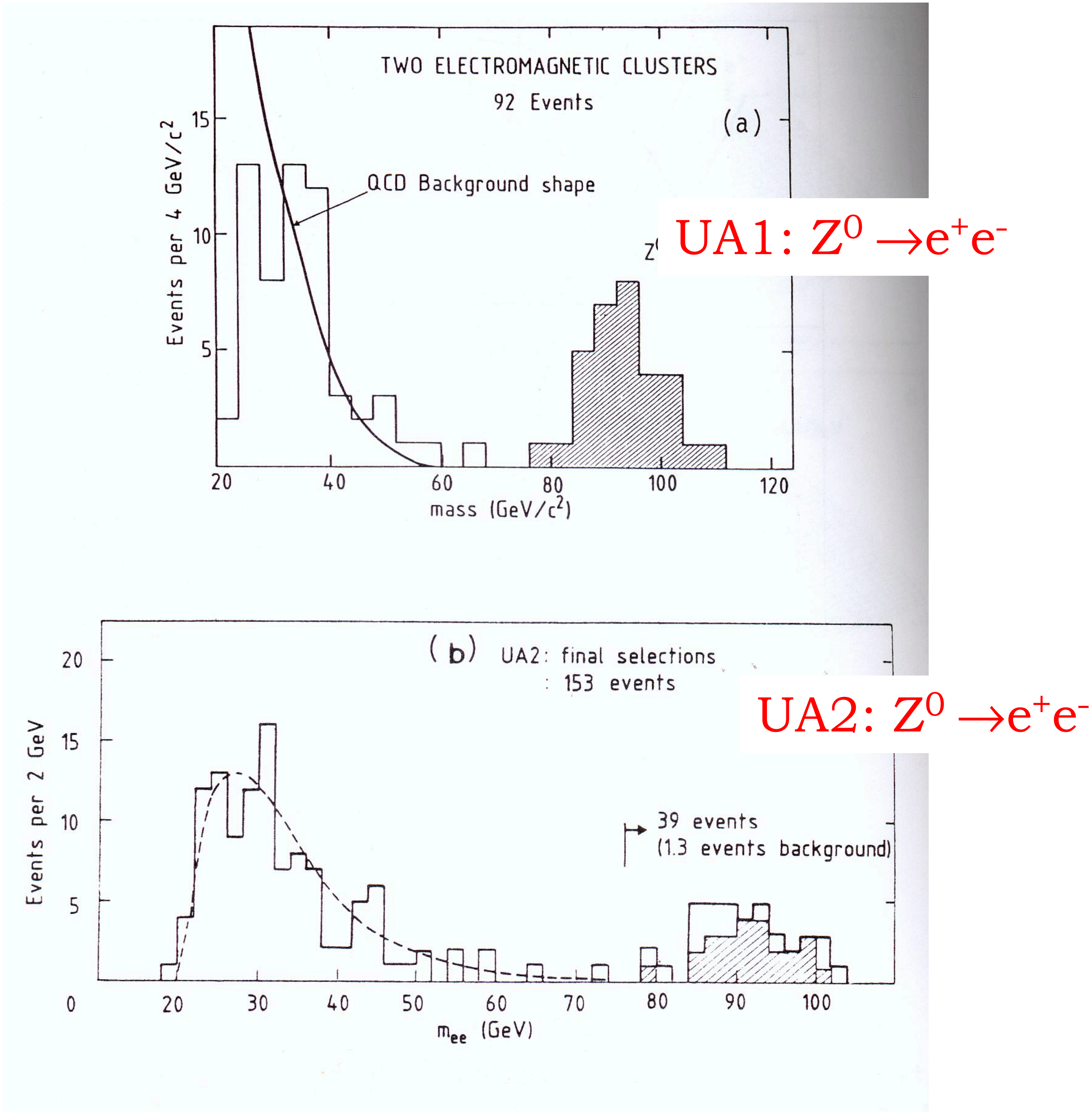
- CERN 1984 collisions $p\bar{p}$
  - $p\bar{p} \to W^+ X^-$ $\qquad W^+ \to \ell^+ \nu_\ell$
  - $pp \to Z^0 X^0$ $\qquad Z^0 \to \ell^+ \ell^-$



$u\bar{d} \to W^+$
$u\bar{u}, d\bar{d} \to Z^0$



- You need 80 to 90 GeV of energy to produce W and Z



continuous symmetries

Continuous symmetry : additive quantum number (conserved)

- space-time symmetry (translation, rotation)

For a unitary transformation T_α one can write $T_\alpha = \exp(-i\alpha Q)$
Q is called the transformation's generator

# of generators = # of parameters in the transformation (eg : 3 generators for the rotation)
The momentum operators are the generators of the translation

- internal symmetry (gauge symmetry : EM) : if global : quantum number conservation (eg baryonic one) ; if local : « appearance » of a vector field (the photon) see later…

Electric charge

- Additive quantum number

Additive quantum number : is a quantity which takes discrete values and the value for a system is equal to the sum of the values of the components of the system

- Analogy with translation

$$S(\alpha) = e^{-i\alpha/\hbar\, Q}$$

Symmetry operator associates to electric charge

Observable : electric charge

- If $S(\alpha)$ commute with H : conservation of electric charge
- In a reaction $\{q_i; i=1....n\} \to \{q_f; f=1....m\}$ on aura $\sum_{i=1}^{n} q_i = \sum_{f=1}^{m} q_f$

- Since all the physical states have a determined charge, the effect of these operators will be to multiply all the wave function by a phase factor

$e^{-i\alpha q/\hbar}$ $\quad q$ Is the Electric charge of the system

Transformation of phase or global gauge transformation

Same other additive quantum numbers are (baryonics, leptonic…). Those are also called internal symmetries

.. Local gauge transformation

- $e^{-i\alpha q/\hbar}$: global gauge transformation →do not modify the Schrödinger eq

- $e^{-iq/\hbar\alpha(\vec{x},t)}$: local gauge transformation if $\psi\left(\vec{x},t\right)$ satisfy Schrödinger eq

$$\boxed{\psi'\left(\vec{x},t\right) = e^{-iq/\hbar\alpha\left(\vec{x},t\right)}\psi\left(\vec{x},t\right) \qquad \text{does not satisfy it !}}$$

- For the charge particles the solution is the following : in presence of an electromagnetic field the Schrödinger eq. is modified such that

$$\frac{1}{2m}\left(-i\nabla + q\vec{A}\right)^2 \psi = \left(i\frac{\partial}{\partial t} + eV\right)\psi \qquad (*)$$

If we define

$$\psi\left(\vec{x},t\right) \rightarrow \psi'\left(\vec{x},t\right) = e^{-iq/\hbar\alpha\left(\vec{x},t\right)}\psi\left(\vec{x},t\right)$$

$$A \quad \rightarrow A' \quad = A + \nabla\alpha$$

$$V \quad \rightarrow V' \quad = V - \frac{\partial\alpha}{\partial t}$$

The eq (*) does not change if

$$\left(\psi,\vec{A},V\right) \rightarrow \left(\psi',\vec{A}',V'\right)$$

It is one of the most important slide in all our lecture !!

- **We could state… that if we impose a local gauge invariance we have to make appearing a field $\left(\vec{A}, V\right)$!!!**

- The existence of a a local invariance $e^{-iq/\hbar\alpha\left(\vec{x},t\right)}$ imply the existence of an electromagnetic interaction (field V, $\vec{A}$) proportional to the charge *q* (the value of q should be determined since is a free parameter of theory !)

Symmetry group $\rightarrow$ interaction (ex : local gauge invariance $\rightarrow \gamma$)

$$\boxed{e^{-iq/\hbar\alpha(\vec{x},t)}} \Longrightarrow (\vec{A},V)$$ of the « classical » electromagnetism

We could state… that if we impose a local gauge invariance we have to make appearing a field. $(\vec{A},V)$

If we quantize this field, it is seen as a particle γ

Quantum Field Theory

The charge is the quantum number conserved by this symmetry transformation
(the value of q has to be determined : free parameter of theory)

U(1)

1 boson / 1 quantum number : the charge

Symmetry group→ interaction (ex : local gauge invariance → γ)

But : unification of electromagnetic and weak interaction

Manifestation of an unique phenomena : electroweak interaction

Electromagnetic current **(γ)** : vector current: γ^{μ}

Neutral current **(Z^0)** : axial and vector

Charged current **($W^{\pm}$)** : should be of this form (V-A) $\gamma^{\mu}(1-\gamma^5)$

?

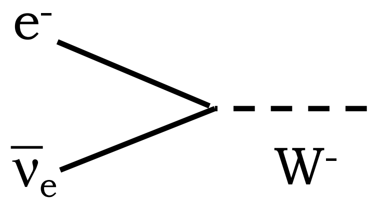
We absorb the term $(1-\gamma^5)$ in the particle spinors: the CC couple only with the left fermions ⇒les CC are like that :

$$j_{\mu}^{-} = \bar{\nu}\gamma_{\mu}\frac{\left(1-\gamma^5\right)}{2}e = \overline{\nu_L}\gamma_{\mu}e_L$$

Parity is put by hands

$$u = u_L + u_R \qquad \Rightarrow j_{\mu}^{elm} = -\bar{e}\gamma_{\mu}e = -\overline{e_L}\gamma_{\mu}e_L - \overline{e_R}\gamma_{\mu}e_R$$

$$j_\mu^- = \overline{\nu_L}\gamma_\mu e_L$$



Left doublet $\chi_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$

$$j_\mu^\pm = \overline{\chi_L}\gamma_\mu\sigma^\pm\chi_L$$

with $\sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $\sigma^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

$$\sigma^\pm = \frac{1}{2}\left(\sigma^1 \pm i\sigma^2\right)$$

I have to create a group

→ structure in families

Looks like the isospin …

With a 3rd current it correspond to σ3 : weak isospin symmetry !

$$\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$j_\mu^3 = \overline{\chi_L}\gamma_\mu\frac{1}{2}\sigma^3\chi_L = \frac{1}{2}\overline{\nu_L}\gamma_\mu\nu_L - \frac{1}{2}\overline{e_L}\gamma_\mu e_L$$

If we continue the parallelism of isospin and we consider the weak hyper-charge which is related to the 3rd component of the weak isospin

$$Q = I_3 + \frac{Y}{2}$$

⇒ Hypercharge weak charge :

$$j_\mu^Y = 2j_\mu^{elm} - 2j_\mu^3 = -2\overline{e_R}\gamma_\mu e_R - \overline{e_L}\gamma_\mu e_L - \overline{\nu_L}\gamma_\mu \nu_L$$

$$j_\mu^{elm} = j_\mu^3 + \frac{1}{2} j_\mu^Y$$

Symmetry group $SU(2)_L \times U(1)_Y$

Weak Isospin(symbol L concerns only left component)

Weak Hyper charge : concern both left and right states

			I	**I_3**	**Q**	**Y**
	doublet L	ν_e	½	½	0	-1
		e_L^-	½	-½	-1	-1
leptons	singlet R	e_R^-	0	0	-1	-2
		u_L	½	½	2/3	1/3
	doublet L	d_L	½	-½	-1/3	1/3
	singlet R	u_R	0	0	2/3	4/3
quarks	singlet R	d_R	0	0	-1/3	-2/3

Same for the other families

In analogy with the $e^{-iq/\hbar\alpha(\vec{x},t)}$

$\mathrm{SU(2)_L}$ $\mathrm{U(1)_Y}$

We have in hands : a « left handed » rotation $U_L = \exp\left[i\frac{\vec{\alpha}\cdot\vec{\sigma}}{2}\right]$ and a phase β :

So :

$$\psi_L \rightarrow e^{iy_L\beta}U_L\psi_L \qquad \psi_R \rightarrow e^{iy_R\beta}\psi_R$$

Left-handed doublet right-handed singlet

Playing the same game as before (local gauge symmetry) :

- $\beta=\beta(x) \Rightarrow$ 1 boson B_μ
- and : $\vec{\alpha} = \vec{\alpha}(x) \Rightarrow$ 3 bosons $W_\mu^{1\,2\,3}$

And we obtain a Lagrangian invariant under SU(2) x U(1) transformation which will contain the interactions !

SU(2)

j_μ^i Coupling to three gauge bosons W_μ^i with coupling g

U(1)

j_μ^Y Coupling to a gauge boson B with coupling $g'/2$

2 independent coupling constants

$$-i\left[\frac{g}{\sqrt{2}} j_\mu^+ \cdot W^{\mu+} + \frac{g}{\sqrt{2}} j_\mu^- \cdot W^{\mu-} + g\, j_\mu^3 \cdot W^{\mu 3} + \frac{g'}{2} j_\mu^Y B^\mu\right]$$

we have :

- a neutral current (W_3^μ) which only has a left-handed component (respecting $SU(2)_L$)
- a neutral current (B^μ) which couples to left-handed and right-handed particles

- we want :
  - the elm current which a left-handed and a right-handed component
  - the neutral current which also has a left-handed and a right-handed component

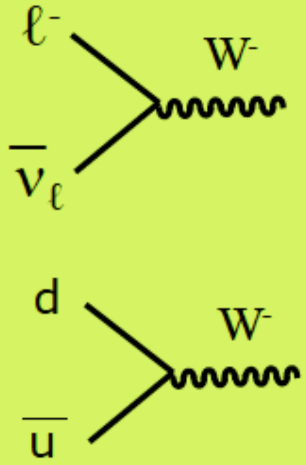
$\Rightarrow$ Define 2 new fields linked to W_3^μ and B^μ :

$$W_3^\mu = \cos\theta_W Z_0^\mu + \sin\theta_W A^\mu$$
$$B^\mu = -\sin\theta_W Z_0^\mu + \cos\theta_W A^\mu$$

The idea is to interpret A_μ as the elm field

We redefined g, g' into **e, g and θ_W** and

$$\Rightarrow \boxed{g\sin\theta_W = e}$$



Charged current

The W± fields

$$L_{CC} = \frac{-g}{2\sqrt{2}} W^{\dagger}_{\mu} \left[\overline{q}_u \gamma^{\mu} (1-\gamma_5) q_d + \overline{\nu}_{\ell} \gamma^{\mu} (1-\gamma_5) \ell \right] + \mathrm{hc}$$

Quark lepton universality
(same coupling)

Neutral current

The Z^0 field

The γ field

$$\Rightarrow \quad L_{NC} = \overline{\nu}_L \gamma_{\mu} \left[\frac{-e}{2\sin\theta_W \cos\theta_W} Z_0^{\mu} \right] \nu_L + \overline{e}_L \gamma_{\mu} \left[\frac{-e}{2\sin\theta_W \cos\theta_W} \left(-1 + 2\sin^2\theta_W\right) Z_0^{\mu} + eA^{\mu} \right] e_L$$

$$+ \overline{e}_R \gamma_{\mu} \left[\frac{e}{2\sin\theta_W \cos\theta_W} \left(2\sin^2\theta_W\right) Z_0^{\mu} + eA^{\mu} \right] e_R$$

SO many predictions with a little number of parameters :

e, g , θ_W

And in fact all the measurement done sofar
all are in agreements with the SM predictions

Problem with the mass scales

remember that

$$\psi(\vec{x},t) \to \psi'(\vec{x},t) = e^{-iq/\hbar\alpha(\vec{x},t)}\psi(\vec{x},t)$$
$$A \to A' = A + \nabla\alpha$$
$$V \to V' = V - \frac{\partial\alpha}{\partial t}$$

Gauge symmetry :

IT WAS ONE OF THE REASON OF USING THE GAUGE INVARIACE AS A SYMMETRY, BECAUSE IT IMPLIES MASS OF THE PHOTON TO BE ZERO

The term $L_M = \frac{1}{2} m_\gamma^2 A^\mu A_\mu$ is not gauge invariant

→ $m_\gamma = 0$

→ but also $m_W = 0$ and $m_Z = 0$

Experimentally : $m_\gamma = 0$ (to a very good extend … 10^{-17} …)

m_W = 80 GeV

m_Z = 91 GeV

In addition, the mass terms for the fermions are of the form : $-m_f \bar{f}f = -m\left(\bar{f}_L f_R + \bar{f}_R f_L\right)$

Which is not gauge invariant …→ $m_f = 0$

Experimentally : $m_e \sim 0.5$ MeV

…

$m_{top} \sim 170$ GeV

→ In our model all the particles are massless … !

Short digression on the mass

$$E^2 = \vec{p}^{\,2} + m^2 \;\rightarrow\; \partial^\mu\partial_\mu + m^2\phi = 0 \;\leftrightarrow\; L = \partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m^2\phi^2 = 0$$

$$(i\gamma^\mu\partial_\mu - m) = 0 \;\leftrightarrow\; L = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi$$

Short digression on the mass

$$E^2 = \vec{p}^{\,2} + m^2 \quad \rightarrow \quad \partial^\mu \partial_\mu + m^2 \phi = 0 \quad \leftrightarrow \quad L = \partial^\mu \phi \partial_\mu \phi - \frac{1}{2} m^2 \phi^2 = 0$$

$$(i\gamma^\mu \partial_\mu - m) = 0 \quad \leftrightarrow \quad L = i\bar{\psi}\gamma_\mu \partial^\mu \psi - m\bar{\psi}\psi$$

$$m\bar{\psi}\psi \quad = m\bar{\psi}(P_L + P_R)\psi = m\bar{\psi}(P_L P_L + P_R P_R)\psi =$$

$$= m[(\bar{\psi}P_L)(P_L\psi) + (\bar{\psi}P_R)(P_R\psi)]\psi \qquad\qquad = m\left(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R\right)$$

The mass should appear in a LEFT-RIGHT coupling

ψ_R : SU(2) singlet

ψ_L : SU(2) doublet

The mass terms are not gauge invariant under

$SU(2)_L \times U(1)_Y$

ψ_R (I=0,Y=-2) leptoni_R

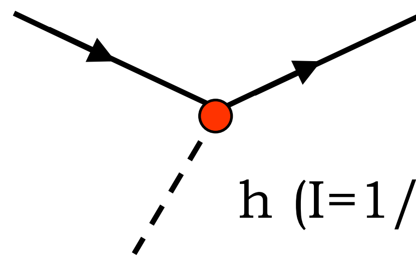
(I=0,Y=-2/3) quark d_R

(I=0,Y=4/3) quark u_R

ψ_L (I=1,Y=-1) leptoni_L

(I=1,Y=1/3) quark d_L

(I=1,Y=1/3) quark u_L



h (I=1/2,Y=1)

Adding a doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad I = \frac{1}{2} \quad Y = 1$$

Yukawa interaction : $\bar{\psi}_L \phi \psi_R$

The way out : the Higgs mechanism (spontaneous symmetry breaking)

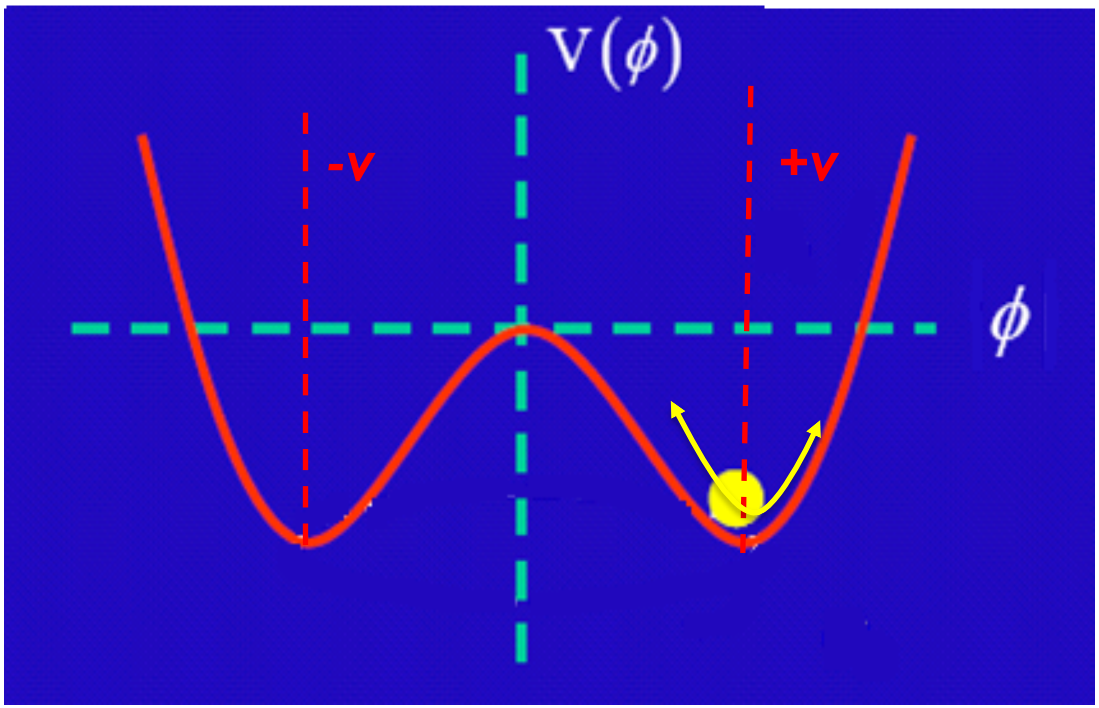
Scalar potential $V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$

"vacuum expectation value"

If $\lambda>0$ and $\mu^2<0$

Degenerate minima : $\phi = \pm v$ with $v = \sqrt{\frac{-\mu^2}{\lambda}}$

$\Phi=0$ is not the minimum

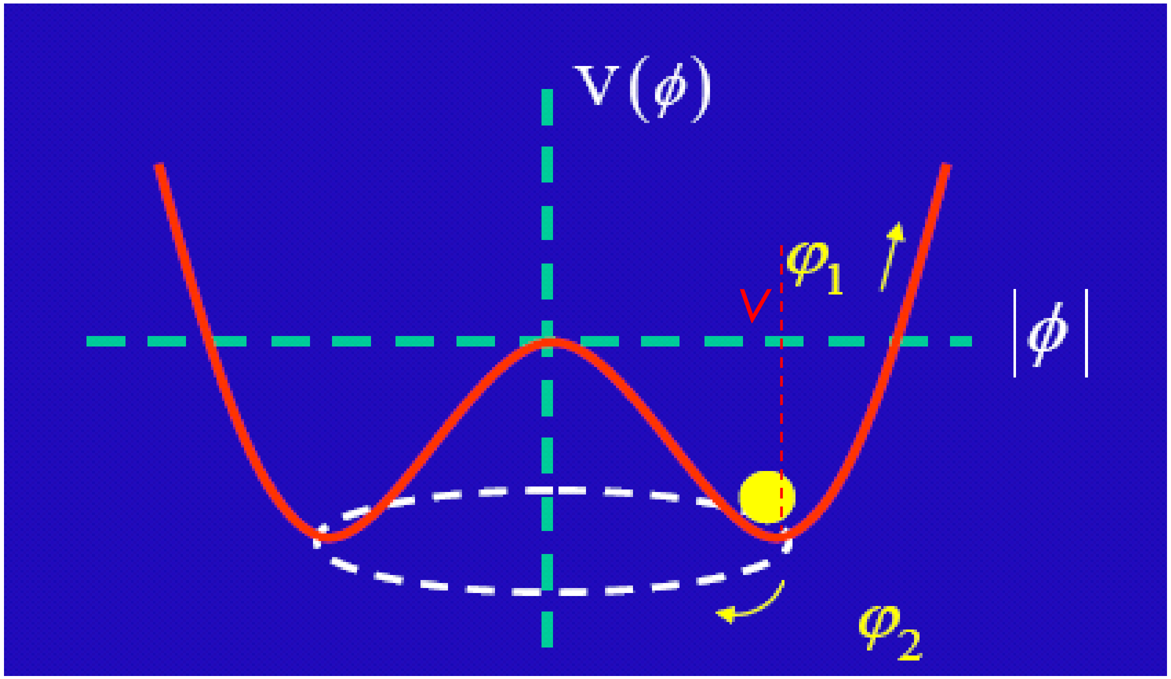


By the choice of the minimum the symmetry is broken

the oscillations around the minimum : the mass of the field

If one uses a complex scalar field $\Phi = 1/\sqrt{2}(\phi_1 + i\Phi_2)$

Degenerate minima : $\phi_1^2 + \phi_2^2 = v$ with $v = \sqrt{\frac{-\mu^2}{\lambda}}$



The Φ_1 field has a mass (just as before)

No corresponding mass term for the Φ_2 field

And in the Standard Model ?

We have just seen that the addition of a well chosen scalar field “modifies” the mass content of the theory

Use a doublet of complex fields

$$\boxed{\phi = \begin{pmatrix} \phi^{+} \\ \phi^{0} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \\ \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4) \end{pmatrix}}$$

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

keep $U(1)_{em}$ invariance

$M_\gamma=0$

Massive weak gauge bosons :

$$M_W = \frac{1}{2} vg$$

$$M_Z = \frac{1}{2}\frac{vg}{\cos\theta_W}$$

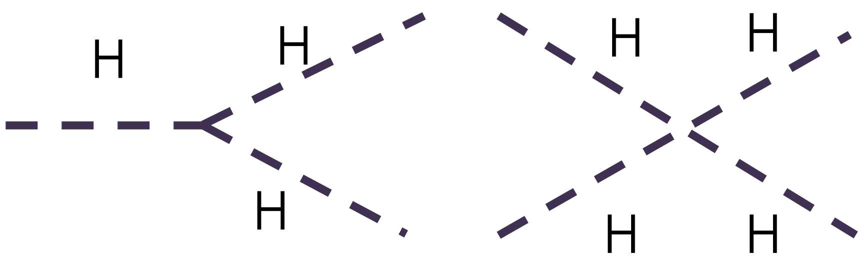
The Higgs boson :

$M_H=\sqrt{-2\mu^2}$ is a free parameter

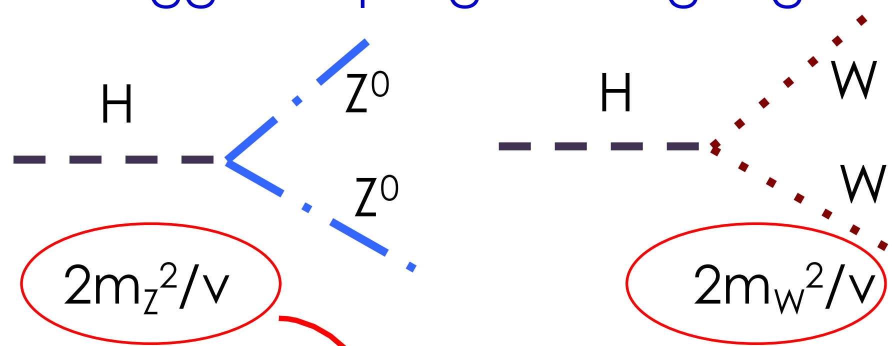
Higgs self-coupling and Higgs couplings to the gauge bosons

These couplings are proportional to the masses

Higgs self coupling



Higgs coupling to the gauge bosons



Couplings proportionnal to the mass of the Higgs

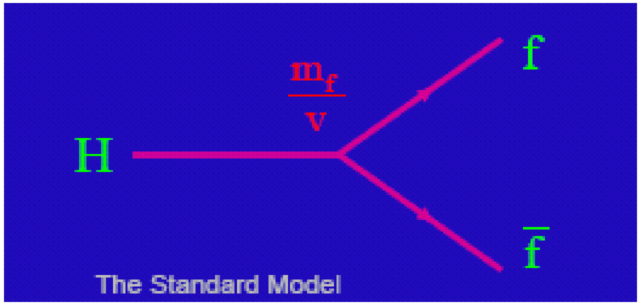
→ **Crucial test of the model**

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{1}{2v^2}$$

v can be computed using the muon decay rate
→ couplings fixed

The Higgs and the fermions :

The fermions masses are free parameters :



The couplings are fixed : m_f/v

Experimental consequence : the Higgs boson will decay preferentially to heavy particles

Note : this is not the most elegant part of the SM.

The Higgs mechanism allows to explain how the **elementary** particles acquire a mass but says nothing about the values

- Complex doublet scalar field : the Higgs field : 3 components absorbed : masses to the W and Z.

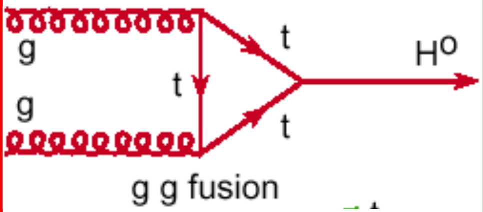
- One remaining component : the Higgs boson

- Higgs field : it is the interaction of the elementary particles with the Higgs field which gives them masses

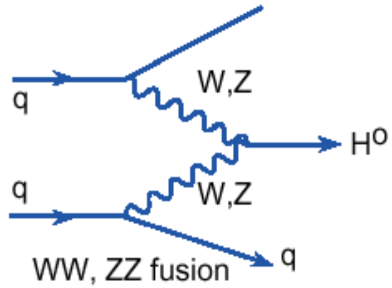
Production (at the LHC)

In the proton : light quarks and gluons → small/no direct coupling to H
→ First produce heavy particles !

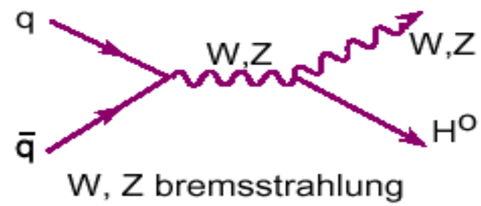
Gluon fusion

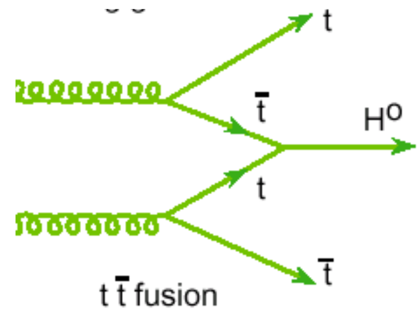


Vector boson fusion



Associated production (W/Z, tt)

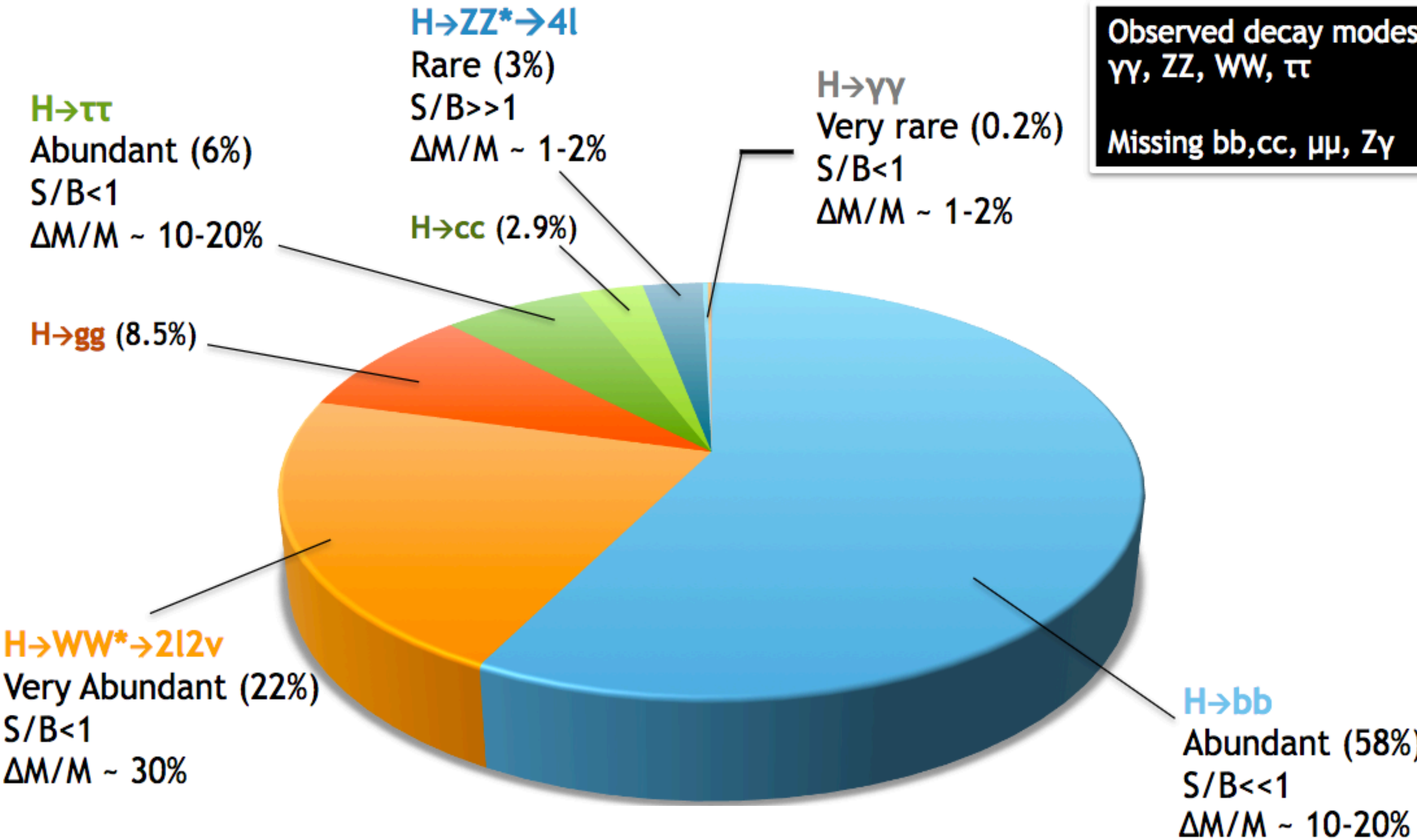




86 %   7 %   5 %   0.6 %

For a 125 GeV H boson

Decay (~~at the LHC~~)



Observed decay modes:
γγ, ZZ, WW, ττ

Missing bb,cc, μμ, Zγ

W,Z : high p_T leptons

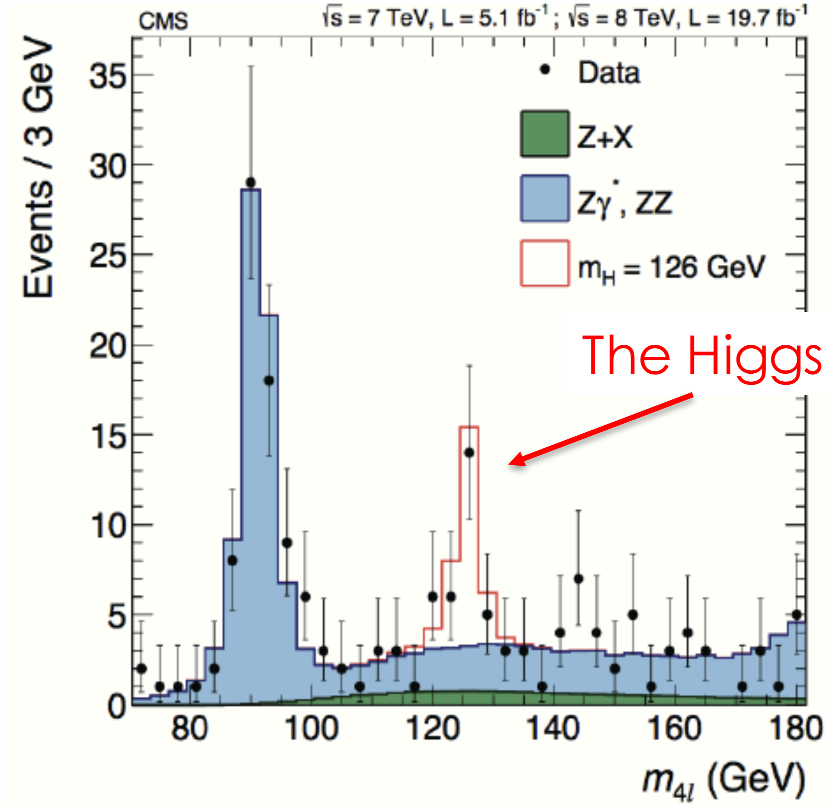
τ : low p_T leptons

bb & ττ : importance of vertex detectors

LABORATOIRE DE L'ACCÉLÉRATEUR LINÉAIRE

The first experimental proposals (LoI) for ATLAS and CMS : 1992

Discovery 20 years latter !



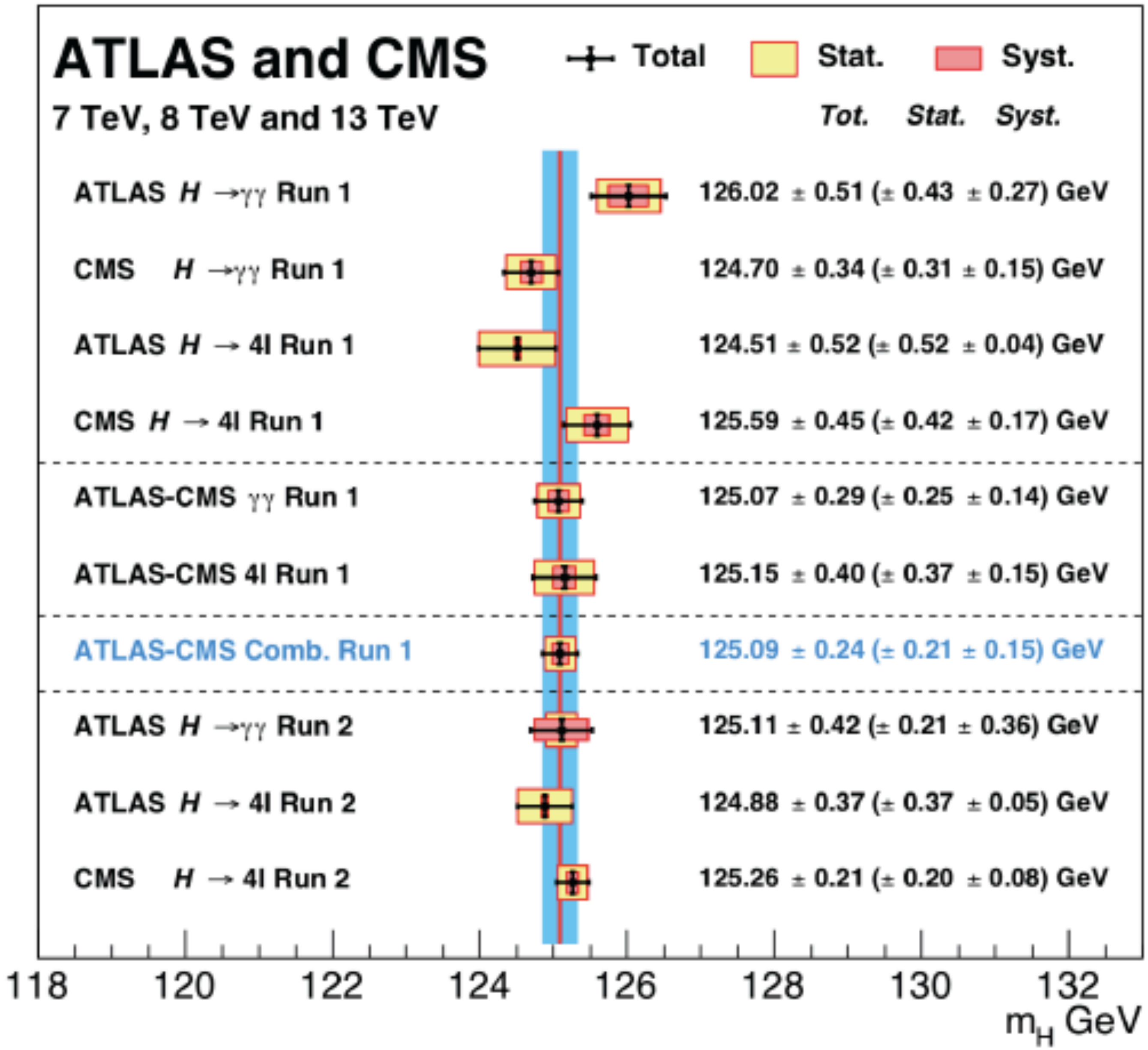
The Higgs a real particle

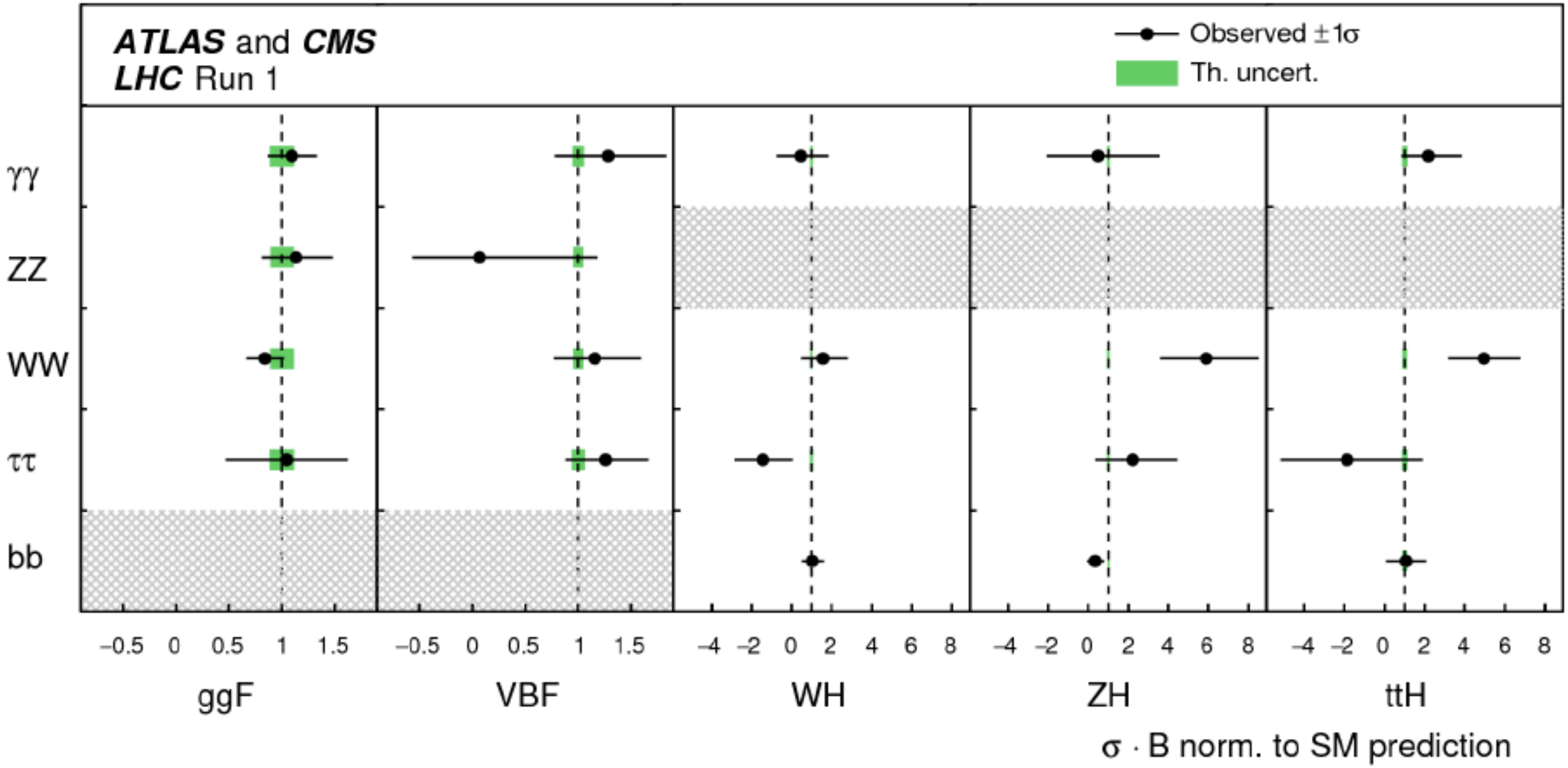
See the presentation from David Rousseau for more details

Nowadays : Higgs boson precision measurements !

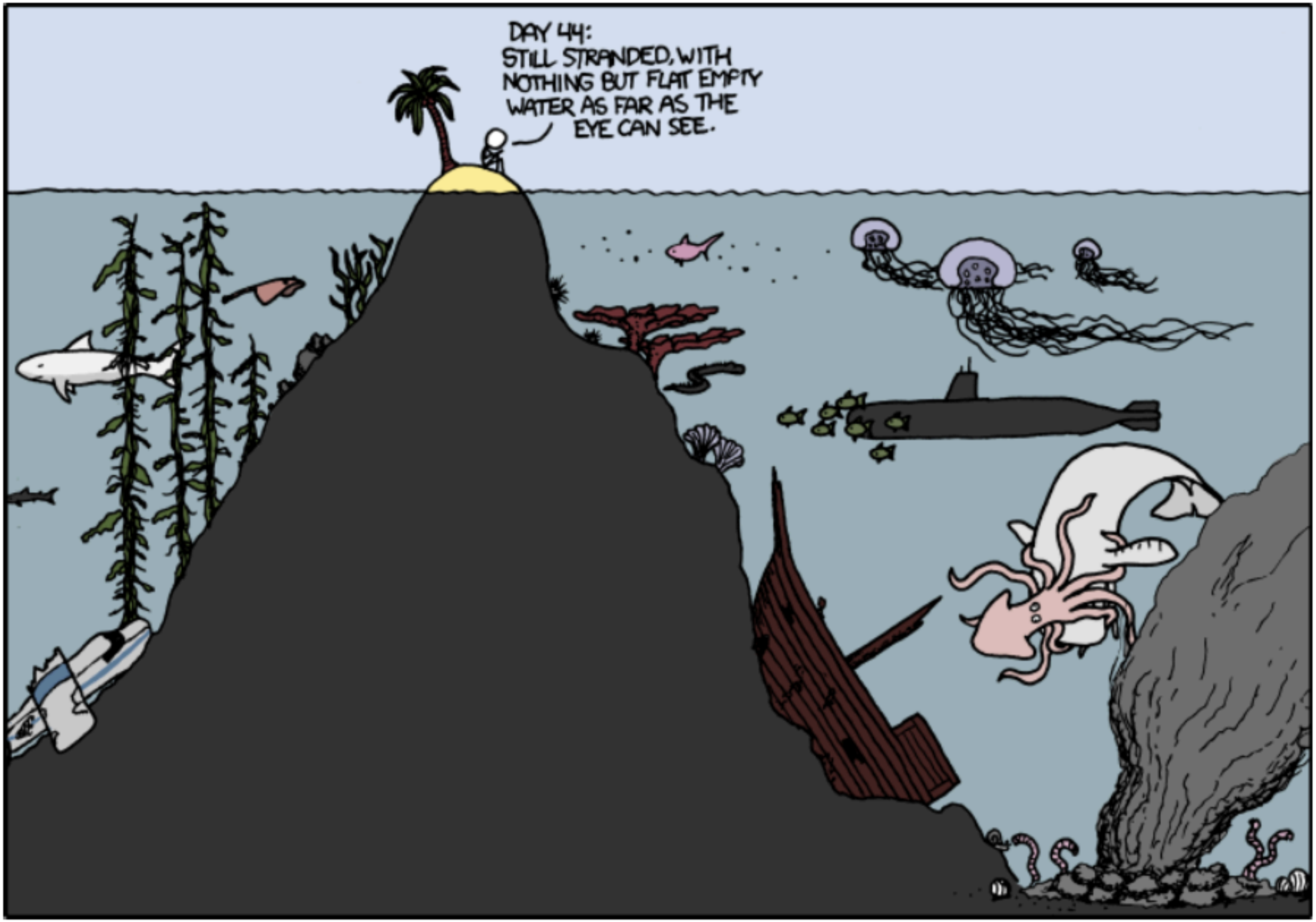
Overall precision : 0.2 %

Marumi Kado @ CERN Summer School 2018




ATLAS and CMS
LHC Run 1
Observed ±1σ
Th. uncert.
γγ
ZZ
WW
ττ
bb
−0.5 0 0.5 1 1.5
−0.5 0 0.5 1 1.5
−4 −2 0 2 4 6 8
−4 −2 0 2 4 6 8
−4 −2 0 2 4 6 8
ggF
VBF
WH
ZH
ttH
σ · B norm. to SM prediction

Up to now it looks like a Standard Model Higgs




THE END
WB