

November 10-16, 2018

An-Najah N. University, Nablus, Palestine

Observational cosmology

- Cosmology basics
- Stochastic background of gravitational waves
- Primordial cosmic strings
- Let's constrain cosmic string models

Cosmology in a nutshell

→ Cosmological principle:

at large scales (>10 Mpc) the universe is **isotropic** and **homogeneous**

→ Distances are measured by integrating the distance element: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$

→ Friedmann-Lemaitre-Robertson-Walker metric in spherical coordinates:

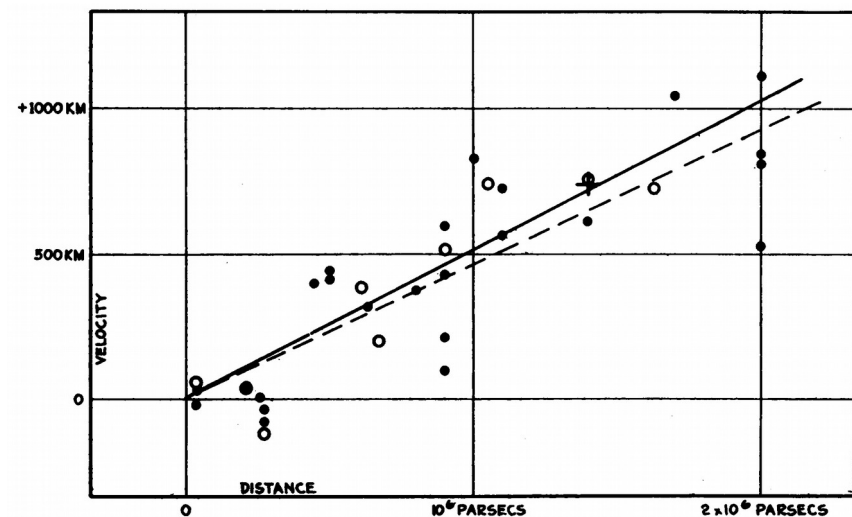
$$ds^2 = dt^2 - a(t) \left[\frac{dr^2}{1 - kr^2} + r^2 \sin(\theta) d\theta + d\phi^2 \right]$$

$a(t)$ = scaling factor (intergalactic distances), normalized such that $a(0) = 1$

k = curvature (-1, 0, 1)

→ FLRW metric in Einstein's equation: $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$

$$\rightarrow H^2(t) = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho + \frac{\Lambda}{3} - \frac{k}{a^2}$$



Densities

Critical density: $\rho_c = \frac{3H_0}{8\pi G}$

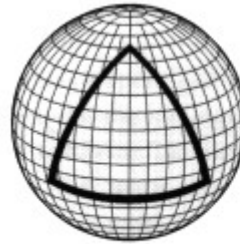
→ matter $\Omega_{mat} = \frac{\rho_{mat}}{\rho_c}$

→ radiation $\Omega_{rad} = \frac{\rho_{rad}}{\rho_c}$

→ cosmological constant $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c}$

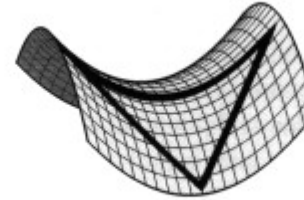
→ total $\Omega_{tot} = \frac{\rho_{tot}}{\rho_c}$

$\Omega_{tot} > 1$



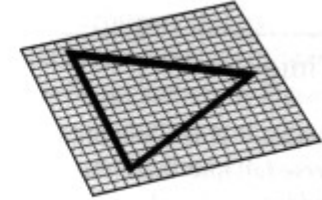
Positive Curvature

$\Omega_{tot} < 1$



Negative Curvature

$\Omega_{tot} = 1$



Flat Curvature

$$H^2(t) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2}$$



Today

$$\Omega_{tot} = 1 + \frac{k}{H_0^2} = \Omega_{mat} + \Omega_{rad} + \Omega_\Lambda$$

State equation

Local conservation of energy $\frac{\partial \rho}{\partial t} = -3(\rho + P)\frac{\dot{a}}{a}$

State equation $P = w\rho$ $\frac{\partial \rho}{\partial t} = -3\rho\frac{\dot{a}}{a}(1+w)$ $\rho = \rho_0 a^{-3(1+w)}$

→ matter $w \approx 0$

$$\rho = \rho_0 a^{-3}$$

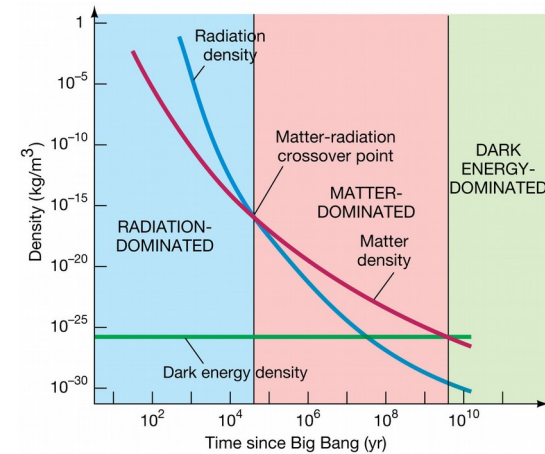
→ radiation $w = 1/3$

$$\rho = \rho_0 a^{-4}$$

→ cosmological constant $w = -1$

$$\rho = \rho_0$$

$$H^2(t) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2}$$



$$H^2(t) = H_0^2 \left[\Omega_{mat} a^{-3} + \Omega_{rad} a^{-4} + \Omega_{\Lambda} \right]$$

$$\Omega_{mat} = 0.308$$

$$\Omega_{rad} = 9.1 \times 10^{-5}$$

$$\Omega_{\Lambda} = 0.692$$

$$H_0 = 67.8 \text{ km/s/Mpc}$$

redshift

A photon is emitted at time t with $\lambda = \lambda_e$

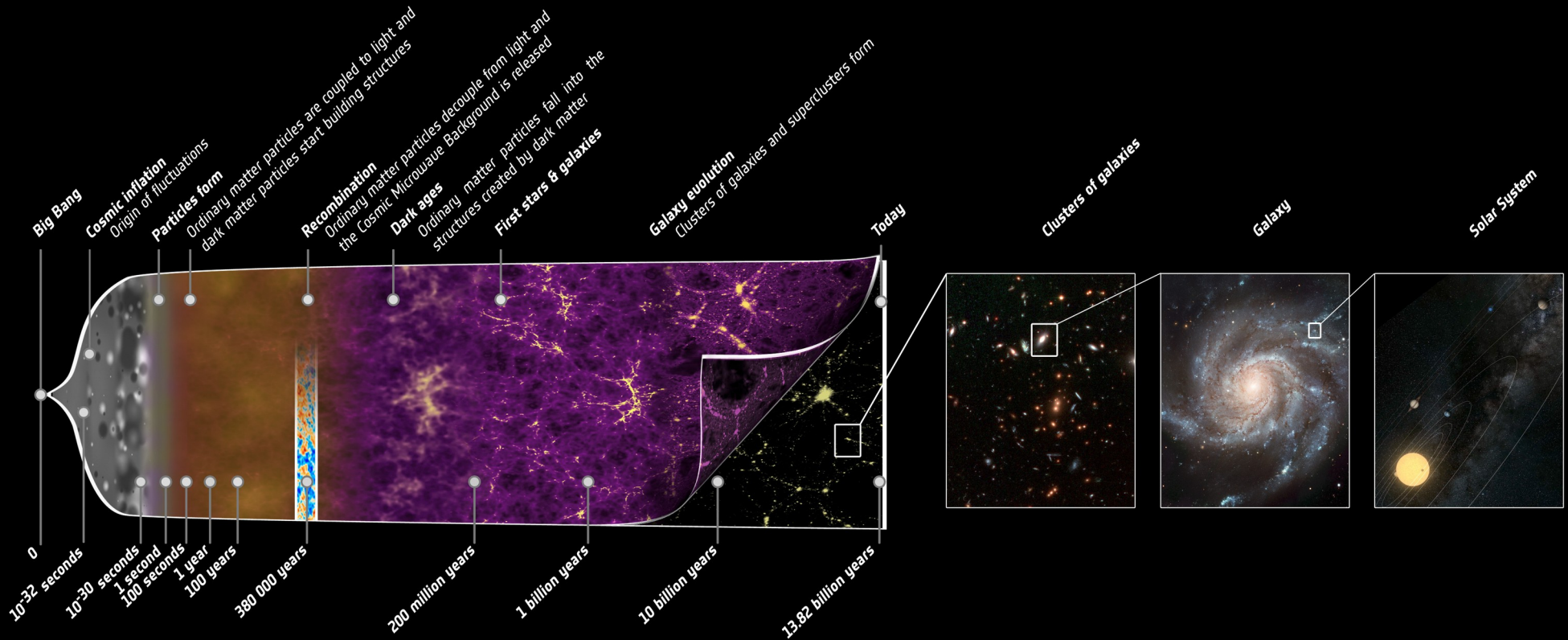
The photon is detected today with $\lambda = \lambda_d$

$$1+z = \frac{\lambda_d}{\lambda_e} = \frac{a(0)}{a(t)} = \frac{1}{a(t)}$$

$$H^2(t) = H_0^2 \left[\Omega_{mat} z^3 + \Omega_{rad} z^4 + \Omega_{\Lambda} \right]$$

Lambda-CDM cosmology \rightarrow standard model for cosmology

History of the universe



Inflation

$$z > 10^{25}$$

Nucleosynthesis

$$z \sim 3 \times 10^8$$

CMB

$$z \sim 1100$$

Dark ages

$$20 < z < 1100$$

Stars and galaxies formation

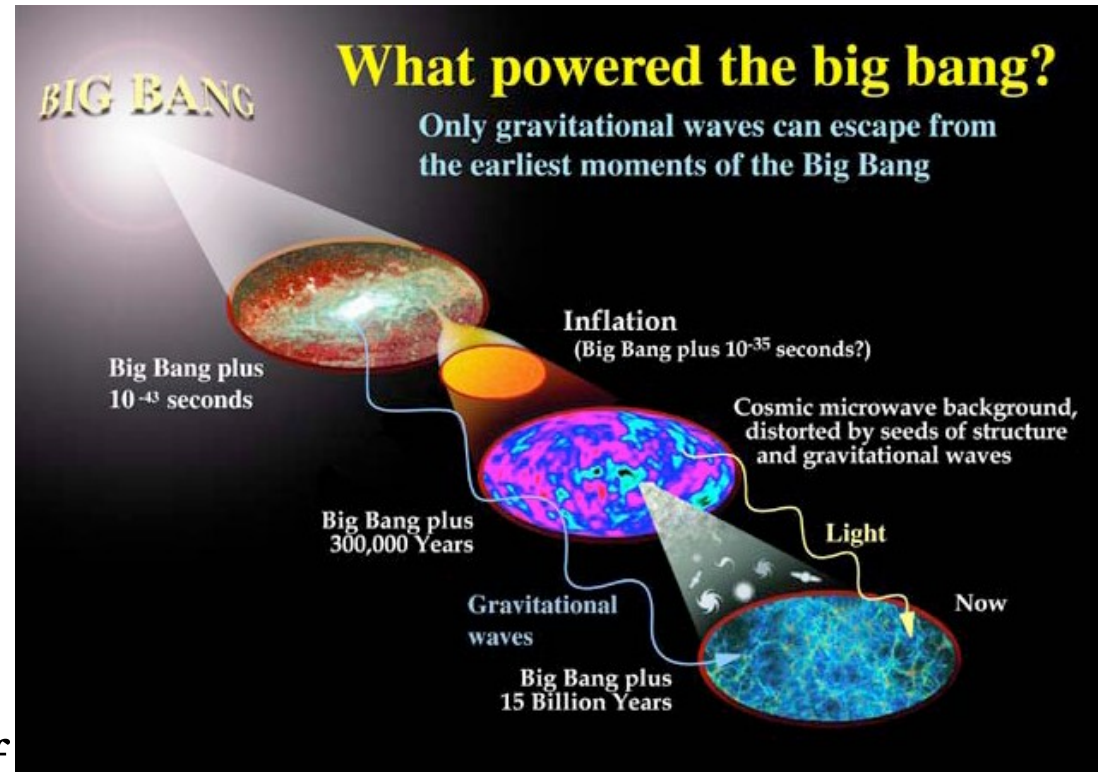
$$z < 20$$

Cosmology

Astrophysics

GW stochastic background

- Incoherent superposition of many unresolved sources.
- Cosmological:
 - Inflationary epoch, preheating, reheating
 - Phase transitions
 - Cosmic strings
 - Alternative cosmologies
- Astrophysical:
 - Supernovae
 - Magnetars
 - Binary black holes
- Potentially could probe physics of the very-early Universe.



$$\Omega_{GW}(f) = \frac{f}{\rho_c} \frac{d\rho_{GW}}{df}$$

GW stochastic background

Assumption : stationary, unpolarized, and Gaussian stochastic background

→ Cross correlate the output of detector pairs to eliminate the noise

$$h_i = n_i + GW_i$$

$$\langle h_1, h_2 \rangle = \langle GW_1, GW_2 \rangle + \underbrace{\langle n_1, GW_2 \rangle}_0 + \underbrace{\langle GW_1, n_2 \rangle}_0 + \underbrace{\langle n_1, n_2 \rangle}_0$$

With $\langle x_1, x_2 \rangle = \int_{-\infty}^{+\infty} \tilde{x}_1^*(f) \tilde{Q}(f) \tilde{x}_2(f) df$

Optimal filter:

$$\tilde{Q}(f) \propto \frac{\gamma(f) \Omega_{GW}(f)}{f^3 S_{n,1}(f) S_{n,2}(f)}$$

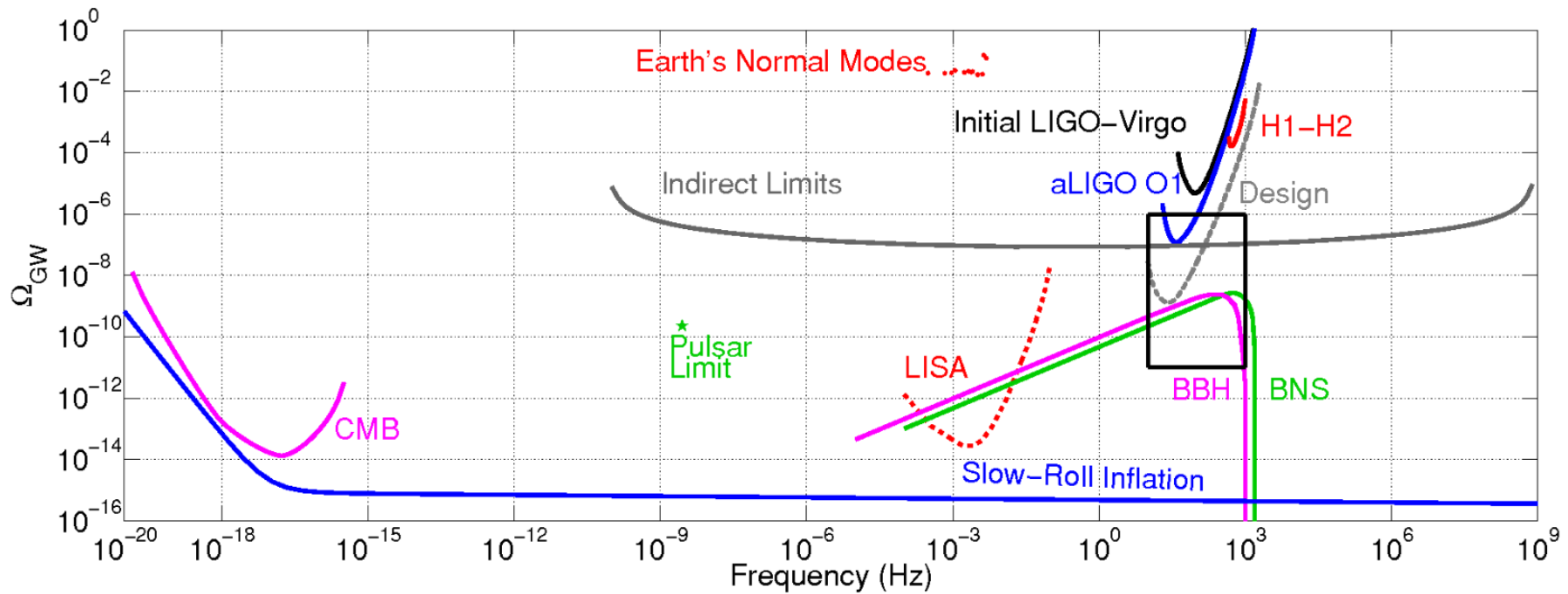
← overlap of antenna pattern
← GW spectrum

$$\Omega_{GW}(f) = \Omega_\alpha f^\alpha$$

↙ ↘
 Detector PSDs

O1 isotropic search, for $\alpha = 0$: $\Omega_{GW}(25 \text{ Hz}) < 1.7 \times 10^{-7}$

GW stochastic background



Use the LIGO upper limit to constrain the parameters of:

- an astrophysical population, e.g. CBC
- an cosmological model, e.g. inflation models

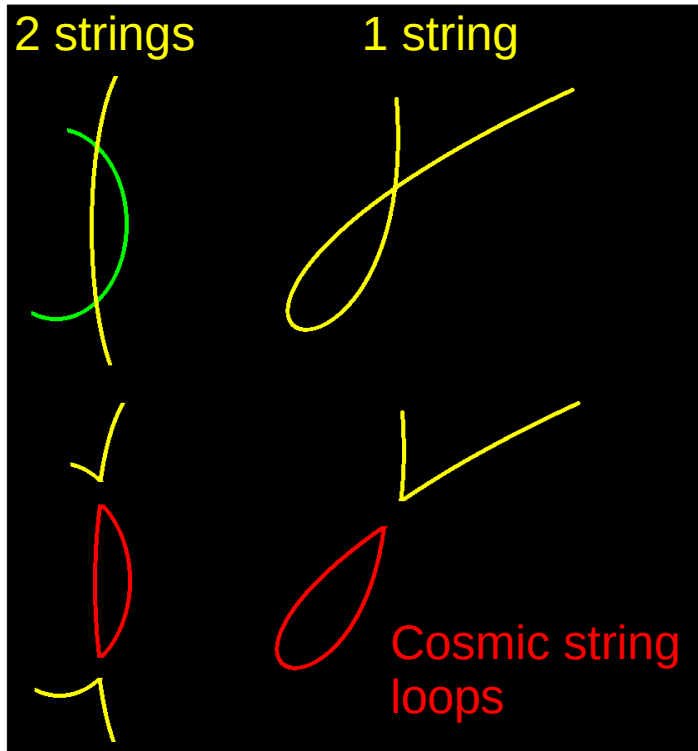
Incoherent superposition of GWs from a population of sources

$$\Omega_{GW}(f) = \frac{4\pi^2}{3H_0^2} f^3 \int_0^{h^*} dh h^2 \int_0^{+\infty} dz \frac{d^2 R}{dz dh}(h, z, f)$$

Cosmic strings

Cosmic strings are 1-dimensional topological defects which may have formed in the early universe.

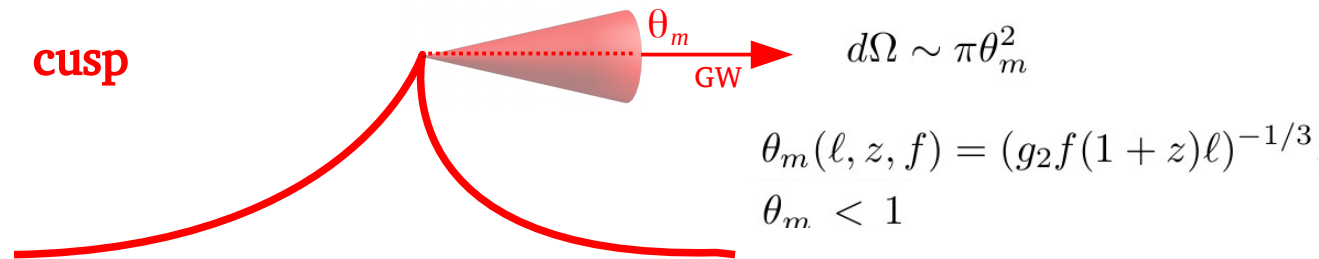
Phase transitions → symmetry breaking → Higgs mechanism → Inflation → topological defects



Loop formation
Loop oscillation → cusps

Models/simulations are used to compute the loop number density:

$$n(l, t)$$



GW waveform:

$$h(\ell, z, f) = A_q(\ell, z) f^{-q}$$

$$A_q(\ell, z) = g_1 \frac{G\mu\ell^{2-q}}{(1+z)^{q-1}r(z)}$$

$q = 4/3$ for cusps

Cosmic string GW rate

$$\frac{d^2 R}{dV(z) dA} (A, z, f)$$

Number density of loops with sizes between l and $l+dl$: $n(l, t) dl$

Period of loop oscillation: $T = l/2$

→ In average we have N cusps produced per loop oscillation

→ number of cusps produced per unit space-time volume by loops of size in dl at time t ?

$$\nu(l, t) dl = ?$$

Cosmic string GW rate

$$\frac{d^2 R}{dV(z) dA} (A, z, f)$$

Number density of loops with sizes between l and $l+dl$: $n(l, t) dl$

Period of loop oscillation: $T = l/2$

→ In average we have N cusps produced per loop oscillation

→ number of cusps produced per unit space-time volume by loops of size in dl at time t ?

$$\nu(l, t) dl = \frac{2N}{l} n(l, t) dl$$

Loop density

$$\Omega_{GW}(f) = \frac{4\pi^2}{3H_0^2} f^3 \int_0^{h^*} dh h^2 \int_0^{+\infty} dz \frac{d^2 R^M}{dz dh}(h, z, f)$$

